term alone. Note that model 1 has this term only.

In Fig. 2, the computed results using model 1 with $C_\phi = 0.4$ are also shown. This value was recommended by Launder et al.[3] in reference to equilibrium shear flows. The computed results using the coefficient $C_\phi = 0.4$ show that the levels of $\overline{u^2}$ increase about 20%, while those of $\overline{v^2}$ decrease about the same amount; this indicates that the redistributive action is reduced about 20% by changing C_ϕ from 0.667 to 0.4. Therefore, the turbulence energy created due to mean strain of the mainstream flow $\overline{u^2}$ is not transferred completely to the normal components $\overline{v^2}$ and $\overline{w^2}$ of the turbulence stresses.

It was generally observed that all the models underpredict the levels of the Reynolds stresses by 10-50%. In order to predict the Reynolds stresses more accurately in recirculating flows, the original models tested for equilibrium shear flows need to be revised. For ease of analysis, model 1 was chosen to refine prediction of the turbulence level. Two computations were performed by using values of 0.2 and 1.2 for C_ϕ. The results are shown in Fig. 3. As depicted in this figure, the small value of coefficient C_ϕ results in appropriate levels of $\overline{u^2}$ but unacceptably low levels of $\overline{v^2}$. On the other hand, the large value of C_ϕ gives reversed results; that is, the levels of $\overline{v^2}$ are substantial but those of $\overline{u^2}$ are too low.

When the flow reattaches to the center cylinder, the flow starts accelerating in the downstream direction, which causes a high mean normal strain as the flow recovers. Thus, the term with $\overline{u^2}\partial U_1/\partial x_1$ in the pressure-strain correlation in the $\overline{u^2}$ equation becomes higher than those in the fully developed flows, which results in significant energy transfer from $\overline{u^2}$ to $\overline{v^2}$. This effect must be suppressed to some extent in the present flow situation, whereas the corresponding component, which balances the energy level of $\overline{v^2}$, is relatively small. Therefore, the redistributive action in the $\overline{v^2}$ equation needs to be promoted more.

For the reasons discussed above, the cases in which the coefficient C_ϕ for $\overline{u^2}$ was decreased to 0.2 and in which coefficient C_ϕ for $\overline{v^2}$ was raised to 1.2 is demonstrated in the same figure. The results obtained with this treatment show much improvement, as indicated in Fig. 3. This observation suggests that the coefficients of the isotropic generation rates for the Reynolds stresses [i.e., the first terms of Eqs. (4-6)] should be adjusted by their values by the effect of the mean strains since the flow patterns are strongly affected by the mean strains. The strain variations are particularly complex in the reattaching shear layers being accompanied by the recirculating flows. In this way the Reynolds stresses can be more accurately evaluated by using such simpler formulations.

Conclusions

The models by Naot et al. and Launder et al. effectively give similar results, and both models are reasonably reliable. The energy redistribution can be improved for reattaching shear flows by taking into account the effects of mean strain.

Acknowledgment

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Reverse Flow Radius in Vortex Chambers

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Nomenclature

A_{in}	= total inlet area
P	= static pressure
Q	= inlet volumetric flow rate
Q_r	= volumetric flow rate of the reverse flow
R_{CF}	= core radius (radius of maximum tangential velocity)
R_E	= exit port radius
R_r	= reverse flow radius
R_0	= radius of the main chamber
V_r, V_θ, V_z	= radial, tangential, and axial velocity components, respectively
Γ	= strength of the vortex
φ	= angle between total velocity vector and tangential velocity component at the inlet

Subscripts

in, F	= properties evaluated at the inlet and on the exit plane, respectively

Introduction

It is well known that when sufficiently large swirl is applied at the inlet of a vortex chamber, the resulting vortex breakdown at the centrally located outlet port is accompanied by a strong reverse flow near the axis of rotation. Recently, it has been experimentally demonstrated that a significant reduction in static pressure drop across the chamber (inlet to outlet) can be obtained by plugging the exit flow.[1] In the present Note, the dimensionless reverse flow radius is determined based on the equations of motion and energy. The reverse flow radius is shown to depend solely on geometrical parameters of the vortex chamber.

Analysis

From the experimental results of Kwok,[2] it can be seen that the static pressure outside the core radius R_{CF} is approximately equal to the ambient (P_a). Inside the core, the static pressure decreases noticeably. Close to the axis of symmetry, the parabolic profile of the pressure suggests a strong influence of the pressure by the solid body rotation. If a funnel-like flow entrainment is assumed to take place, the reverse flowfield must resemble that of Fig. 1. At $z = 0$, the radial velocity component will be approximately zero while a large average axial velocity component must be present. If the static pressure at $r = R_{CF}$ is P_a, it will be below the ambient in the interval (0, R_{CF}). From the radial momentum equation,

$$P_F(r) = (\rho m^2/2)(r^2 - R_{CF}^2) + P_a \tag{1}$$

where

$$m = \Gamma/2\pi R_{CF}^2$$

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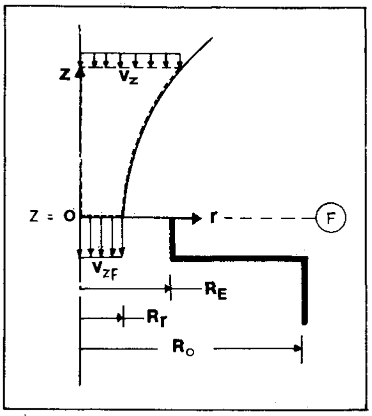

Fig. 1 Entrained flow at the outlet of the vortex chamber.

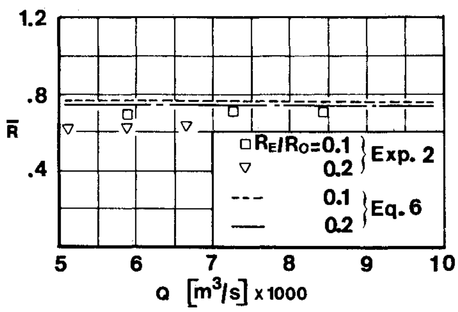

Fig. 2 Reverse flow radius vs inlet volumetric flow rate of different contraction ratios (R_E/R_0).

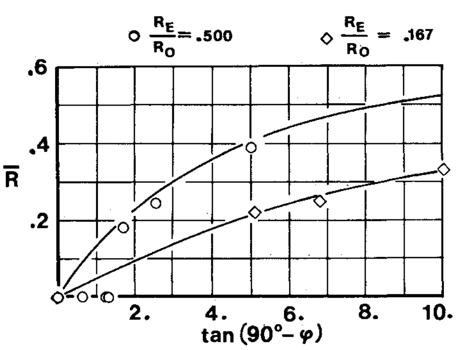

Fig. 3 Reverse flow radius vs inlet swirl (○, ◇: experimental points from Ref. 4).

The energy equation for the control volume shown by the dashed lines in Fig. 1 is

$$\lim_{z\to\infty}\iint_{A_z}\left(P_z+\rho\frac{q_z^2}{2}\right)V_z\mathrm{d}A_z=\iint_{A_F}\left(P_F+\rho\frac{q_F^2}{2}\right)V_{zF}\mathrm{d}A_F+\dot{F} \quad (2)$$

As $z\to\infty$, $P_z\to P_a$, and $V_z\to 0$, then Eq. (2) becomes

$$2\pi\int_0^{R_r}\left[P_a+\rho\frac{m^2}{2}(r^2-R_{CF}^2)+\rho\frac{q_F^2}{2}\right]V_{zF}r\mathrm{d}r=P_aQ_r-\dot{F} \quad (3)$$

where $q_F^2=V_{zF}^2+(mr)^2$ and $\dot{F}$ is the rate of energy lost to viscous dissipation in bringing the fluid from "infinity" into the chamber. To evaluate $\dot{F}$, the detail fluid structure inside the control volume is required. In this analysis, $\dot{F}$ is neglected. Performing the integration in Eq. (3), and after a series of algebraic manipulations, one obtains

$$V_{zF}=m\sqrt{R_{CF}^2-R_r^2} \quad (4)$$

Continuity gives

$$V_{zF}=Q_r/\pi R_r^2$$

and Eq. (4) yields

$$Q_r=mR_E^3\bar{R}^2\sqrt{X_{CF}^2-\bar{R}^2} \quad (5)$$

where $\bar{R}=R_r/R_E$, X_{CF} is the dimensionless core size (R_{CF}/R_E) and is determined by the following equation[3]:

$$X_{CF}^4(2\ell n X_{CF}-1)+2X_{CF}^2[(a+1)-\ell n X_{CF}]-1=0$$

where

$$a=[(A_{in}/A_0)/(R_E/R_0)\cos\varphi]^2 \text{ and } A_0=\pi R_0^2$$

Study of Eq. (5) shows that 1) when $\bar{R}\to 0$, then $Q_r\to 0$, and 2) when $\bar{R}\to X_{CF}$, then $Q_r\to 0$.

Therefore, an $\bar{R}$ must exist in (0, X_{CF}), where Q_r attains a maximum value. This is given by

$$\bar{R}=0.816X_{CF} \quad (6)$$

If indeed $\bar{R}$ is given by Eq. (6), it must not be a function of the inlet volumetric flow rate. This deduction is evident from Fig. 2. The agreement of the present theory with the experimental results of Ref. 2 is reasonable.

On the other hand, $\bar{R}$ must vary with the inlet swirl number [$V_{\theta in}/V_{rin}$ or tan(90 deg $-\varphi$)]. This effect is presented in Fig. 3. Experimental evidence[4] shows that a "dimp" is first developed in the axial velocity profile at the exit. The "dimp" grows as the inlet swirl number is enhanced and, eventually, if the swirl strength is sufficiently large, develops into a reverse flow. Equation (6) indicates that the reverse flow will be present even with small swirl. In this respect, the present theory does not predict the physical phenomena. The reason lies with the assumptions made earlier. However, when the reverse flow has been established, Eq. (6) predicts the real case well.

Concluding Remarks

The dimensionless reverse flow radius has been found to depend solely on the chamber geometrical parameters. In the presence of a reverse flow at the outlet, the present theory approximates the physical solution reasonably well.

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